RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. SECOND SEMESTER TAKE-HOME TEST/ASSIGNMENT, JULY 2020
FIRST YEAR (ARREAR)

Date : 06/07/2020 **MATHEMATICS (General)**
Time : 11am - 1pm **Paper : II** Full Marks : 40

Instructions to the Candidates

- Write your Name, College Roll no, Subject and Paper Number on the top of the Answer Script and on the text body of the mail.
- Read the instructions given at the beginning of each paper/group/unit carefully.
- Only handwritten (by blue/black pen) answer-scripts will be admissible.
- Each paper/group must be answered in a single booklet.
- All the pages of your answer scripts must be numbered serially by hand.
- In the last page of your answer-scripts, please mention the total number of pages written so that we can verify it with that of the scanned copy of the scripts sent by you.
- For an easy scanning of the answer scripts and also for getting better image, students are advised to write the answers in single side and they must give a minimum 1 inch margin at the left side of each paper.
- After the completion of the test, you should scan the entire answer scripts by using Clear Scan : Indy Mobile App /other Scanner devices and make a file in your own name and send or share them as a PDF file to [redacted]

Group-A (2 dimensional geometry)

Answer any two questions from question number 1-3: **[4 $\times$ 2 =8 marks]**

1. Prove that the equation to the straight lines through the origin each of which makes an angle α with the straight line $y = x$ is $x^2 - 2xy\sec 2\alpha + y^2 = 0$. [4]
2. Prove that if the straight line $\lambda x + \mu y + v = 0$ touches the parabola $y^2 - 4px + 4pq = 0$, then $\lambda^2 q + \lambda v - p\mu^2 = 0$. [4]
3. Reduce the equation, $3x^2 + 2xy + 3y^2 - 16x + 20 = 0$, to its canonical form and determine the nature of the conic represented by it. [4]

Group - B (Vector Algebra)

Answer any two from the question numbers 4-6. **[4 x 2 = 8 marks]**

4. Show that the vectors $\hat{i} + 2\hat{j} + \hat{k}$, $\hat{i} + \hat{j} - 3\hat{k}$ and $7\hat{i} - 4\hat{j} + \hat{k}$ are mutually perpendicular. [4]
5. If the volume of a tetrahedron be 2 units and three of its vertices be $A(1, 1, 0)$, $B(1, 0, 1)$, $C(2, -1, 1)$, then find the locus of the fourth vertex. [4]

6. Find the vector equation of a straight line passing through the point $3\hat{i} - \hat{j} + \hat{k}$ and parallel to the vector $2\hat{i} + \hat{j} - 4\hat{k}$. [4]

Group - C (Differential Calculus)

Answer any two questions from question numbers 7-9. **[6 x 2 = 12 marks]**

7. (a) Find the maximum value of $\left(\frac{1}{x}\right)^x$. [3]

   (b) Give examples of two real sequences $\{x_n\}$ and $\{y_n\}$, where $\{x_n\}$ diverges to ∞ and $\{y_n\}$ diverges to $-\infty$, but the sequences $\{x_n + y_n\}$ and $\{x_n - y_n\}$ both diverge to $+\infty$. [3]

8. (a) Check the convergence of the series [3]

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2\sqrt{n}}$$

   (b) Show that between any two roots of $e^x \cos x = 1$, there exists at least one root of $e^x \sin x - 1 = 0$. [3]

9. (a) Find the rectilinear asymptotes of the following function [4]

$$f(x) = \frac{e^x + e^{-x}}{e^{-x} - e^x}$$

   (b) Find the maximum and minimum values of the function $f(x,y) = x^2 - xy + y^2$ on the quarter circle $x^2 + y^2 = 1, x \geq 0, y \leq 0$. [2]

Group - D (Integral Calculus I)

Answer any two from the following question nos. 10-12. **[3 x 2 = 6 marks]**

10. Evaluate [3]

$$\int \frac{11\cos x - 16\sin x}{2\cos x + 5\sin x} dx$$

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11. Show that [3]

$$\int_0^{\frac{\pi}{2}} \sqrt{\sin x} \cos^5 x dx = \frac{64}{231}$$

.

12. Evaluate

$$\int_1^3 (x^2 + 5x) dx$$

by the method of summation. [3]

Group-E (Ordinary differential equation 1)

Answer any two from question nos. 13-15. **[3 × 2 = 6 marks]**

13. Solve: $(y^2 + 2x^2y)dx + (2x^3 - xy)dy = 0$. [3]

14. Solve:

$$\frac{dy}{dx} = \frac{x + y + 4}{x - y - 6}.$$

[3]

15. Solve: $(x^2y^2 + xy + 1)ydx + (x^2y^2 - xy + 1)xdy = 0$. [3]

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